Monte Carlo European Option Pricing

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Monte Carlo Simulation

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Monte Carlo Simulation

Monte Carlo (MC) simulation is a computerized mathematical technique used to understand how a complex model responds to randomly generated variables. It is one of the methods for representing a complex financial model.

The technique relies on the generation of a large number of random samples in which the randomly generated variables interact with the structure of the simulation to provide a probability distribution of possible outcomes.
Advantages and Disadvantages

Advantages:

- Simple, easy to implement
- Unrestricted choice of functions for implementation

Disadvantages:

- The output is only as good as the inputs, which may be erroneous
- Not a good representation when the underlying probability distributions are skewed
- Behavioral aspects of finance are not factored in
Applications

The Monte Carlo simulation technique is used widely in disparate fields such as finance, project management, engineering, research and development, insurance, and so on and so forth.

In finance, the Monte Carlo method is used to value financial instruments (such as options, fixed income instruments), portfolios, and investments by simulating the various sources of uncertainty that affect their values and then determining the expected value from the distribution of the generated outcomes.
Diagram
Monte Carlo European Option Pricing Diagram

Step 1: Simulation Parameters
- Expected annual return: \( r \)
- Annual volatility: \( \sigma \)
- Number of periods: \( n \)
- Number of simulations: \( m \)
- Current underlying price: \( S_0 \)
- Dividend yield: \( q \)
- Time to expiration: \( T \) (in years)
- Risk-free rate: \( r_f \)
- Strike price: \( K \)

Step 2: MC Simulation of Underlying
- 1st simulation
  - \( S_1 \) at period 1
  - \( S_2 \) at period 2
  - \( S_3 \) at period 3
  - ... 
  - \( S_T \) at period \( n \)
- 2nd simulation
  - \( S_1 \) at period 1
  - \( S_2 \) at period 2
  - \( S_3 \) at period 3
  - ... 
  - \( S_T \) at period \( n \)
- 3rd simulation
  - \( S_1 \) at period 1
  - \( S_2 \) at period 2
  - \( S_3 \) at period 3
  - ... 
  - \( S_T \) at period \( n \)
- m-th simulation
  - \( S_1 \) at period 1
  - \( S_2 \) at period 2
  - \( S_3 \) at period 3
  - ... 
  - \( S_T \) at period \( n \)

Step 3: Option Pricing
- Option Price at 1st simulation
- Option Price at 2nd simulation
- Option Price at 3rd simulation
- Option Price at m-th simulation

Average Option Price
Present Value of Option Price

\( r_f \) is relevant to the option expiration time horizon.
Steps
MC European Options Pricing Steps

**Step 1**: estimate and collect simulation parameters/inputs

**Step 2**: simulate underlying paths for user-specified “time to expiration”

**Step 3**: aggregate and assess the outputs from simulated paths by computing the average option price
Step 1: Simulation Parameters

- Expected annual return: $r$
- Annual volatility: $\sigma$
- Number of periods: $n$
- Number of simulation: $m$
- Current underlying price: $S_0$
- Dividend yield: $q$
- Time to expiration: $T$
- Risk-free rate: $rf$
- Strike price: $K$
Step 2: Simulating Underlying Paths

Method 1: Arithmetic Brownian motion (discrete form)

\[ S_{t+\Delta t} = S_t + r \cdot S_t \cdot \Delta t + \sigma \cdot S_t \cdot Rnd \cdot \sqrt{\Delta t} \]

Method 2: Geometric Brownian motion (continual form)

\[ S_{t+\Delta t} = S_t \cdot e^{(r - \frac{1}{2} \sigma^2) \cdot \Delta t + (\sigma \cdot Rnd \cdot \sqrt{\Delta t})} \]

Notes:

- **Rnd** is a random number from standard normal N(0, 1)
- \( \Delta t = T/n \)
- Assume the underlying is paying no dividends.
Example of Simulated Paths
Step 3: Option Pricing

For each simulated path (from 1 to m):

- Call = Max (0, $S_T - K$)
- Put = Max (0, $K - S_T$)

European option price:

$$Present\ Value\ (PV)\ of\ Option\ Price = \frac{1}{m} \sum_{i=1}^{m} \frac{Call}{Put} \ast e^{-rf \ast T}$$
Practice
Think you grasp the concept of the MC European option pricing? Try to code this in Excel VBA, and/or R Studio!

Check the results (for consistency): Calculate the option prices with the same input parameters using Black-Scholes formula and see if the results converge.
Black-Scholes Model

\[ C = SN(d_1) - Ke^{-rT}N(d_2) \]

\[ d_1 = \frac{\ln(S/K) + (r + \frac{1}{2} \sigma^2)T}{\sigma \sqrt{T}} \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]

\[ P = Ke^{-rT}N(-d_2) - SN(-d_1) \]

or, \[ P = C + Ke^{-rT} - S \] (Put-Call Parity)