Value at Risk

CME Group Foundation Business Analytics Lab

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Value at Risk (VaR)

Variance-Covariance Method

Delta-Gamma VaR

Simulation Method

Stress Testing & Worst Case Scenario
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Financial Risks

Two broad categories:

1. Credit risk: risks associated with the reference credit such as default on loans and changes in credit rating.
2. Market risk: risks affecting broad sectors of the economy such as interest rates, exchange rates, and commodity prices.
Value at Risk (VaR)

Value at Risk (VaR) is a statistical technique used to measure and quantify the level of financial risks within a firm or investment portfolio over a specific time frame.

VaR is measured by assessing the amount of potential loss, for a chosen confidence level and time horizon.

It is simply a statement of probable loss. Statement example: There is 5% probability that a firm will lose more than $1 million in one day.
Application

- VaR can be applied to specific positions or portfolios as a whole or to measure firm-wide risk exposure.
- Commonly used by investment and commercial banks to determine the extent and occurrence ratio of potential losses in their institutional portfolios.
- Now its use has spread to non-financial firms, even commonly used by regulators to determine capital reserve requirements of financial institutions.
- For example, financial institutions like banks use VaR modeling to determine whether they have sufficient capital reserves in place to cover losses.
Pros and Cons

Pros:

- Simplified assumptions: normality, constant correlations among risk factors.
- A popular risk management tool gives management a bird’s eye view of risks.
- It can easily incorporate many sources of risk.
Pros and Cons

Cons:

- Simplified assumptions make the estimation easier but have flaws.
- Using a sample path of prices to compute VaR might not reliably reflect the future.
- Statistics pulled from a period of low volatility may understate the potential risk magnitude.
- Under the assumption of Normal Distribution, in the event of fat tail, the risk is underestimated (sometimes extremely).
- Deviations from normality are commonplace.
Estimating Methods

**Parametric:**
- Variance-covariance: calculate the variance of the portfolio based on the variances of each asset in the portfolio and the correlation between risk factors.
- Delta-Normal VaR (linear instruments), Delta-Gamma VaR (non-linear instruments).

**Non-parametric:**
- Historical simulation: use historical data to model behavior of risk factors instead of assuming risk factors normality.
- Monte Carlo simulation: simulate the potential market shocks and uses mathematical modeling to predict future shocks.
- Complementary: stress testing, worst case scenario analysis.
Variance-Covariance Method
Single Asset VaR

Recall the z-standard normal distribution: \( z = \frac{x-\mu}{\sigma} \), \( x = z\sigma + \mu \)

VaR general form: \( \text{VaR} = V(\mu + \sigma z) \)

<table>
<thead>
<tr>
<th>Prob(x&lt;z)</th>
<th>0.1%</th>
<th>0.5%</th>
<th>1.0%</th>
<th>2.5%</th>
<th>5.0%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>z</td>
<td>-3.090</td>
<td>-2.576</td>
<td>-2.326</td>
<td>-1.960</td>
<td>-1.645</td>
<td>-1.282</td>
</tr>
</tbody>
</table>

Example: \( V=1000 \), Daily returns follow \( N(0, 0.01^2) \), \( c = 95\% \).

\[ 1 \text{ day VaR} = (-1.645 \times 0.01 + 0) \times 1000 = -16.45 \]

Meaning: there is 5\% probability that the asset will lose more than $16.45 next day.
Charting VaR

Area Left of z-score
Portfolio VaR: Two Assets with Weights \( w_1 \) and \( w_2 \)

\[
\begin{align*}
\mu_p &= w_1 \mu_1 + w_2 \mu_2 \text{ (mean)} \\
\sigma_p^2 &= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho \text{ (variance)} \\
\rho &= \frac{\text{cov}(r_1, r_2)}{\sigma_1\sigma_2} \\
\text{VaR} &= V_p(\mu_p + \sigma_p z)
\end{align*}
\]
Portfolio VaR: N Assets

\[ \sigma_p^2 = w^T \times Cov \times w = \begin{bmatrix} w_1 & \ldots & w_n \end{bmatrix} \begin{bmatrix} \sigma_{11} & \ldots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \ldots & \sigma_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} \]

Where: “Cov” is the Var-Covar matrix and “T” means transpose

\[ \mu_p = w^T \mu = \begin{bmatrix} w_1 & \ldots & w_n \end{bmatrix} \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix} \]

\[ \text{VaR} = V_p(\mu_p + \sigma_p z) \]
Delta-Gamma VaR
Risk Factors and Portfolio Sensitivity

A set of $m$ risk factors (e.g., interest rate): $x = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$.

Behavior of these factors could be measured based on absolute changes, percentage changes, standard returns, or logarithmic returns: $\partial x = \begin{bmatrix} \partial x_1 \\ \vdots \\ \partial x_m \end{bmatrix}$.

The value of the portfolio ($P$) with $k$ instruments:

$$P(x(t)) = \sum_{k=1}^{n} P_k(x(t)).$$
The sensitivities (greeks) of the instruments can be used to approximate the effect of the changes of the risk factors.

- For a linear instrument, the effective delta (first-order derivative) is a good calculation.
- For non-linear instruments, higher order terms like gamma (second-order derivative) must be included.

Delta sensitivity:  
\[
\Delta = \begin{bmatrix}
\Delta_1 \\
... \\
\Delta_m
\end{bmatrix}, \text{ where } \Delta_i = \frac{\partial P}{\partial x_i}
\]

For a given factor \( x_i \):  
\[
\Delta P \approx \Delta_i dx_i.
\]
So for all risk factors:  
\[
\Delta P \approx \Delta^T dx, \text{ note that delta weighted sum of market factors changes.}
\]
Delta-Normal VaR (linear instruments)

The portfolio deltas with respect to market factors are represented by: $\Delta = \begin{bmatrix} \Delta_1 \\ \vdots \\ \Delta_m \end{bmatrix}$, where $\Delta_i = \frac{\partial P}{\partial x_i} = \sum_{k=1}^{n} \frac{\partial P_k}{\partial x_i}$ (delta sensitivity)

The linear combination of normally distributed market factor changes is also normally distributed. Change in portfolio can be approximated by: $\Delta P \approx \Delta^T dx$ (delta weighted)

Standard deviation is calculated by: $\sigma_p = \sqrt{\Delta^T C \Delta}$, where $C$ is the covariance matrix $\text{cov}(x_i, x_j) = E((x_i - \mu_i)(x_j - \mu_j))$

Portfolio VaR: $\text{VaR} = \alpha \sigma_p \sqrt{T}$ (when mean is 0)
Delta-Gamma VaR (non-linear instruments)

Gamma vector is given by:

\[
\Gamma = \begin{bmatrix}
\Gamma_1 \\
\vdots \\
\Gamma_m
\end{bmatrix}, \text{ where } \Gamma_i = \frac{\partial^2 P}{\partial x_i^2} \text{ (gamma sensitivity)}
\]

The cross terms in the second order changes: \( \Gamma_{ij} = \frac{\partial^2 P}{\partial x_i \partial x_j} \).

The \( \Gamma \) matrix of the portfolio: \( \Gamma_{ij} = \frac{\partial^2 P}{\partial x_i \partial x_j} = \sum_{k=1}^{n} \frac{\partial^2 P_k}{\partial x_i \partial x_j} \)

\[
\Delta P \approx \Delta^T dx + \frac{1}{2} dx^T \Gamma dx
\]
Simulation Method
Historical Simulation

AKA non-parametric VaR, uses historical data to predict changes in risk factors instead of assuming risk factors normality. **Steps:**

1. Identify a pool of historical data time series to capture the risk factors over a specified time period.
2. Select a set of historical scenarios to simulate changes in risk factors.
3. Portfolio revaluation: value the portfolio based on the change of risk factors assuming history will most likely repeat itself over a time frame.
4. P&L Distribution: $V_p(t+1) - V_p(t)$, repeat this in the data period and create a distribution by ranking all the outcomes by P&L.
5. Portfolio VaR: choose a confidence level, and the value at that percentile in the distribution represents the portfolio VaR.
Monte Carlo Simulation

Use Monte Carlo method to simulate the market risk factor scenarios. **Steps:**

1. Generate an equation to model the behavior of the risk factors.
2. Simulate the behavior of the risk factors for a large number of times.
3. Portfolio revaluation: value the portfolio based on each path of the risk factors.
4. P&L Distribution: $V_p(t + 1) - V_p(t)$, repeat this in the data period for each path and create a distribution by ranking all the outcomes by P&L.
5. Portfolio VaR: choose a confidence level, and the value at that percentile in the distribution represents the portfolio VaR.
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Flowchart Diagram

Historical Simulation:

1. Historical Time Series
2. Market Scenario
3. Portfolio Revaluation
4. P&L Distribution
5. Portfolio VaR

Monte Carlo Simulation:

1. Monte Carlo Simulation
2. Market Scenario
3. Portfolio Revaluation
4. P&L Distribution
5. Portfolio VaR
Stress Testing & Worst Case Scenario
Stress Testing

Stress testing estimates how the portfolio would have performed under some extreme market conditions. For example a five standard deviation move in a market variable in a single day or a particular extreme market scenario like the crash of 2008. This is almost impossible under normal distribution assumption.

In theory happens almost once in 7,000 years. In reality it happens close to once every 10 years or less!
Worst Case Scenario (WCS) Analysis

Worst case scenario analysis is a complementary measure to VaR. It estimates VaR in a large number of times (may use Monte Carlo Simulation). The worst cases are taken from each time and are incorporated into one distribution of the worst case scenario outcomes, then from that distribution compute the worst case scenario for the portfolio. The mean value is usually used, sometimes 1% and 5% levels are used to gauge risks under extreme market conditions.